

Multiscale fluctuations in nuclear response

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Abstract

The nuclear collective response is investigated in the framework of a doorway picture in which the spreading width of the collective motion is described as a coupling to more and more complex configurations. It is shown that this coupling induces fluctuations of the observed strength. In the case of a hierarchy of overlapping decay channels, we observe Ericson fluctuations at different scales. Methods for extracting these scales and the related lifetimes are discussed. Finally, we show that the coupling of different states at one level of complexity to some common decay channels at the next level, may produce interference-like patterns in the nuclear response. This quantum effect leads to a new type of fluctuations with a typical width related to the level spacing.

1 Introduction

Lifetimes and damping mechanisms are general questions in physics. In quantum mechanics, lifetimes are directly related to the width of the considered states through the Heisenberg relation. Lifetimes of simple systems, such as single particle states are often related to quantum tunnelling. In many-body systems, the damping of excitations appears more complex since it can involve many different processes. This is in particular the case for collective excitations of nuclei for which several origins for the observed width have been discussed so far[1, 2, 3]. On the first hand, a fragmentation of the

collective response into several collective states is expected, leading to the so-called Landau damping. The collective motion can also be directly coupled to the continuum of escaping states[4, 5], giving rise to the escape width. Finally, the collective strength can decay toward the compound nucleus configurations. This spreading width is often viewed in a doorway picture as the coupling to more and more complex states. For instance, a collective excitation built from particle-hole excitations can be coupled to two-particle two-hole states through the residual two-body interaction [6]. Again, those states might themselves decay toward three-particle three-hole states because of two-body collisions. After many such steps up in complexity, this process eventually ends in the chaos of compound nucleus states[7, 8, 9, 10](which may, in general, induce a fine-structure in the response of the nucleus[11]). In such a picture, collective excitations of many-body systems exhibit a large variety of time scales for the decay mechanism going from the short lifetime of the collective motion associated to a width of several MeV, to the long lived compound nucleus states with a typical width of the order of few eV or even less.

Correlations are a well known tool to measure lifetime of quantal system. Indeed, when the level spacing is small compared with the typical width of each individual level the observed spectrum is known to exhibit Ericson fluctuations[12]. In such a case, excitation spectra presents fluctuations with a correlation width characteristic of the average width of the overlapping states. As far as the collective response is concerned, because of the various scales involved in the decay process, one may wonder about the characteristics of the expected fluctuation pattern. Since the compound nucleus states can be considered as the true eigenstates of the many-body system, one would predict that the observed fluctuations should be characteristic of the associated lifetime. However, these fluctuations are over a so small scale that most experiments are unable to detect them. On the other hand, collective states have a large width and indeed poor resolution experiments exhibit resonant structure with several MeV width. Improving upon the resolution one thus expects to uncover more and more detailed structures directly related to the different levels of complexity in the damping mechanism. The Landau spreading can be first studied. Then, according to the commonly admitted picture, one may look at the coupling to the two-particle two-hole states using a finer coarse graining, and so on and so forth down to the compound nucleus scale. Is it possible to observe such a multiscale structure in

the fluctuation pattern, from the collective mode toward different levels of complexity down to the compound nucleus chaos? In such a context, the response function may present more and more detailed structure when the resolution is improved, so one may even ask himself about a possible fractal nature of the response function. In this case, are the fluctuations self-similar or not? And so which interpretation can be given to the observed fluctuations as a function of the resolution in used?

However, the above discussion might well be too simple. Indeed, the robustness of the various complexity levels against the effects of the residual interaction, has not been investigated. Even if from the classical point of view a typical path toward chaos may go through a sequence of bifurcations, there is no guarantee that, in quantum mechanics, the coupling of a regular collective motion to an ensemble of chaotic states follows this path. The simple picture of a hierarchy of more and more complex doorway states is typical of a perturbative approach, but in such large matrix, one strong element (compared with the typical level spacing) may be enough to change the spectrum and the properties of many eigenstates: i.e., at some point, the perturbative approach may break down. Moreover, many quantum effects can also be expected such as the possibility of interferences [13, 14, 15] or the indirect coupling of doorway states through their decay at higher orders in perturbative expansions.

In this article we would like to present a critical discussion of the various questions raised above. The possibility of having multiscale fluctuations is first illustrated in section II. Since, it might be difficult to disentangle various scales in a fluctuating spectrum, we present in section III a brief discussion of the various methods for extracting these scales. Finally, in section IV, we demonstrate through analytical derivations and numerical calculations that fluctuations may also occur due to quantum interferences.

2 Formalism and results

Let us first briefly recall some known results about damping mechanism of doorway states and about the fluctuations of cross sections. In such a way, we will make clear the concepts and notations used in the article.

2.1 Ericson Fluctuations in doorway processes

The collective response can be tested by applying an external field to the nucleus. If we assume that the ground state $|0\rangle$ is excited through a time-dependent field $\lambda \hat{D} (e^{-iEt} + e^{+iEt})$, where \hat{D} is a hermitian operator. The linear response theory tells us that the state of the system is¹[16]

$$|\Psi(t)\rangle = |0\rangle + \lambda \sum_{\mu} |\mu\rangle \left(\frac{\langle \mu | \hat{D} | 0 \rangle}{(E - E_{\mu}) + i\Gamma_{\mu}/2} e^{-iEt} - \frac{\langle \mu | \hat{D} | 0 \rangle}{(E + E_{\mu}) - i\Gamma_{\mu}/2} e^{+iEt} \right) \quad (1)$$

where we have assumed that each eigenstate $|\mu\rangle$ of the system has a finite lifetime² $\tau_{\mu} = 1/\Gamma_{\mu}$. Using the doorway notations $\hat{D}|0\rangle \equiv |Coll\rangle$ and performing a Fourier transform of the observation $\langle \Psi(t) | \hat{D} | \Psi(t) \rangle$, we get the corresponding spectral response which reads for positive energies

$$R(E) = \sum_{\mu} \left(\frac{|\langle \mu | Coll \rangle|^2}{(E - E_{\mu}) + i\Gamma_{\mu}/2} - \frac{|\langle \mu | Coll \rangle|^2}{(E + E_{\mu}) + i\Gamma_{\mu}/2} \right) \quad (2)$$

Since the first term is dominant we can write $R(E) \simeq \sum_{\mu} |\langle \mu | Coll \rangle|^2 / (E - E_{\mu} + i\Gamma_{\mu}/2)$.

The associated strength function is given by:

$$S(E) = -\frac{1}{\pi} \text{Im}(R(E)) \simeq \sum_{\mu} \frac{\frac{\Gamma_{\mu}}{2\pi} |\langle \mu | Coll \rangle|^2}{(E - E_{\mu})^2 + \Gamma_{\mu}^2/4} \quad (3)$$

When the various states $|\mu\rangle$ are well separated, i.e. when the average distance ΔE between two states is much larger than the individual width Γ_{μ} , the strength $S(E)$ presents isolated peaks with a typical width Γ_{μ} . When the states are strongly overlapping, one can still extract information about the individual widths by looking at the fluctuations known as Ericson fluctuations[12]. Indeed, the overlap $O_{\mu} = |\langle \mu | Coll \rangle|^2$ can be separated into two parts

- an averaged part $\bar{O}(E_{\mu})$ which is a smooth function of E_{μ} . In the case of a resonance, this averaged part often take the shape of a gaussian or

¹in all the paper we used $\hbar = 1$

² In the following, the states $|\mu\rangle$ are considered as eigenstates of an effective hamiltonian. In the first part of this article the coupling of these degrees of freedom to the rest of the system will be simply taken into account as a finite lifetime Γ_{μ} . This is valid only for nonoverlapping decay channels as discussed in refs. [13, 14] and in the last chapter.

a lorentzian centered on the collective energy with a width interpreted as the width of the doorway state[17] ;

- a fluctuating one $\delta O_\mu = O_\mu - \bar{O}(E_\mu)$.

Then the response can also be splitted into two components (see appendix 1) . On the one hand, it contains an averaged strength function $\bar{S}(E)$ proportional to \bar{O} and on the other hand remains a fluctuating term $\delta S(E) = S(E) - \bar{S}(E)$ associated with δO_μ . These fluctuations still contain information about the averaged width $\bar{\Gamma}$ as it was demonstrated by Ericson[12]. To measure fluctuations, we can define the autocorrelation function of the strength distribution as

$$C(E) = \frac{1}{\delta E} \int_{E_0}^{E_0+\delta E} dE' \delta S(E') \delta S(E' + E) \quad (4)$$

where δE is the averaging interval. If we assume random correlations between the overlap fluctuation, i.e. $\delta O_\mu \delta O_\nu = \delta O_\mu^2 \delta_{\mu\nu}$, we get the following correlation

$$C(E) = \frac{2}{\pi \Delta E} \frac{\overline{\delta O^2}}{E^2 + \bar{\Gamma}^2} \quad (5)$$

where ΔE is the averaged level spacing. This relation shows that the width of the autocorrelation function is directly related to the typical width of the individual states.

In figure (1), two cases are displayed: in the left part, we illustrate a case with $\bar{\Gamma} \ll \Delta E$, which leads to the fragmented response (in some cases, this is comparable to the Landau spreading), whereas in the right part, we show a typical Ericson case with $\bar{\Gamma} \gg \Delta E$. On top of this figure, we show the strength function³. As discussed in section 3, instead of performing the autocorrelation technique on the strength function, it appears more convenient to perform it on the first derivative of the strength function. Indeed, the autocorrelation on the strength function is particularly suitable when it is applied to a rather flat average spectrum. However, in general because of the global variation of the strength, an average strength has to be removed from the distribution in order to extract the fluctuations[18]. In the case of a resonant average response, we have observed that the width extracted from

³It should be noticed that, since the energy units can be re-scaled, we have chosen arbitrary units to display the various strength functions.

the autocorrelation procedure is sensitive to the method used to define this average (fitting procedure, averaging interval). This might introduce some spurious fluctuations. A way to remove it, is to consider the derivative of the studied spectrum. The derivative of the strength function emphasizes its fine structure and avoid the ambiguities on the averaged strength distribution. In the middle of figure (1), we show the derivative of the strength function and the associated autocorrelation in the bottom. In both cases the width of the autocorrelation is directly proportional to the width of the individual states. Although the two considered strength function are different, we can observe in figure (1) that the associated autocorrelation functions are almost identical. This comes directly from the fact that the autocorrelation applied to the derivative is only sensitive to the smallest scale in the spectra and ignores long-range correlation. In this figure, since we have imposed the same small fluctuations scale in both examples, the two autocorrelation happen to be very similar showing that this technique could be applied on a fragmented strength or a Ericson fluctuation case with the same success.

2.2 Damping of Collective States

In order to get a deeper insight into the fluctuation pattern in the collective response and into the information which can be extracted from their study, we can now consider the microscopic mechanism responsible for this damping. The coupling of the collective motion to the compound nucleus states leads to an internal mixing while the relaxation to the continuum is a true external decay channel. In reference [19], the effect of the continuum was carefully investigated but no particular assumption on the internal degrees of freedom is made. In the present paper we will mainly focus on the complexity of the internal mixing. In particular, we will introduce a hierarchy of degrees of freedom and couplings.

2.2.1 Modelisation of the Doorway Mechanism

Let us first introduce a doorway state $|Coll\rangle$ associated with an unperturbed energy $\hat{H}_0 |Coll\rangle = E_{Coll} |Coll\rangle$. This state is in general not an eigenstate of the total hamiltonian but is coupled to more complexe states. Very often it is possible to introduce a hierarchy of complexity between those states. In particular, giant resonances are assumed to decay toward two-particle two-hole states which are themself coupled to three-particle three-hole states which

are damped by more complex states. Such a mechanism can be described by considering a basis which can be formed into a hierarchy of states $|Coll\rangle$, $\{|i_1\rangle\}$, $\{|i_2\rangle\}$, ... at unperturbed energies $\hat{H}_0 |i_n\rangle = E_{i_n} |i_n\rangle$. These states allows the definition of more and more complex Hilbert spaces $\mathcal{E}_0 \subset \mathcal{E}_1 \subset \mathcal{E}_2 \subset \dots$ where \mathcal{E}_0 contains only $|Coll\rangle$ while \mathcal{E}_n includes all the states up to the n^{th} level, $|i_n\rangle$. We can now introduce a hierarchy of residual interactions \hat{V}_1 , \hat{V}_2 , ... where the interaction \hat{V}_n couples the states of the space \mathcal{E}_{n-1} to the states $\{|i_n\rangle\}$. Then, the diagonalization of the Hamiltonian

$$\hat{H}_n = \hat{H}_0 + \hat{V}_1 + \dots + \hat{V}_n \quad (6)$$

produces eigenstates $|\mu_n\rangle$

$$|\mu_n\rangle = c_{\mu_n, Coll}^n |Coll\rangle + \sum_i c_{\mu_n, i}^n |i\rangle \quad (7)$$

with an energy E_{μ_n} . In the following we will truncate the hierarchy at various level and discuss the fluctuations properties of the associated strength.

2.2.2 The escape width Γ^\dagger as a coherent decay of the states μ

At a given level of complexity, the states μ have an infinite lifetime. However, in nuclei, eigenstates are coupled to the continuum leading to a finite lifetime Γ_μ for each state. This decay to the continuum is also linked to the escape width Γ^\dagger of the collective mode. Indeed, introducing the continuum states $|k\rangle$ and a coupling with the states μ through a residual interaction $\Delta\hat{V}$, we can estimate the width Γ_μ which is related to the transition matrix elements $|\langle k | \Delta\hat{V} | \mu \rangle|^2$ from the states $|\mu\rangle$ to the continuum states $|k\rangle$. Using the wave function (7), we get

$$|\langle k | \Delta\hat{V} | \mu \rangle|^2 = \left| c_{\mu, Coll} \langle k | \Delta\hat{V} | Coll \rangle + \sum_i c_{\mu, i} \langle k | \Delta\hat{V} | i \rangle \right|^2 \quad (8)$$

Since $c_{\mu, Coll}$ and the various $c_{\mu, i}$ are not correlated, the previous expression can be evaluated as an incoherent sum $|\langle k | \Delta\hat{V} | \mu \rangle|^2 \simeq |c_{\mu, Coll}|^2 |\langle k | \Delta\hat{V} | Coll \rangle|^2 + \sum_i |c_{\mu, i}|^2 |\langle k | \Delta\hat{V} | i \rangle|^2$, showing that, as far as the decay to the continuum is concerned, taking advantage of the Fermi golden rule, we can write

$$\Gamma_\mu = |c_{\mu, Coll}|^2 \Gamma^\dagger + \sum_i |c_{\mu, i}|^2 \Gamma_i^\dagger \quad (9)$$

Here we have introduced the direct coupling of the collective states to the continuum: the escape width $\Gamma^\uparrow \propto |\langle k | \Delta \hat{V} | Coll \rangle|^2$. Γ_μ can thus be split into two parts: a collective one $\Gamma_\mu^{Coll} = |c_{\mu, Coll}|^2 \Gamma^\uparrow = O_\mu \Gamma^\uparrow$, which is distributed according to a lorentzian shape (cf eq. (11)), and an individual coupling to the continuum $\Gamma_\mu^\uparrow = \sum |c_{\mu, i}|^2 \Gamma_i^\uparrow$ which is not expected to present any particular structure. Since the $|c_{\mu, Coll}|^2$ are very small, the Γ_μ might be much smaller than Γ^\uparrow . This is actually the case since typical values are the eV for Γ_μ and the hundreds of keV for Γ^\uparrow . Note that, if we take $|c_{\mu, Coll}|^2 \propto 1/N$ where N is the number of states coupled to $|Coll\rangle$: $N \simeq \Gamma_{Coll}/\Delta E$ with ΔE the average spacing between two states μ , we see that the collective contribution to Γ_μ is proportional to $\Gamma^\uparrow/N = \Gamma^\uparrow \Delta E/\Gamma_{Coll}$ which is smaller than Γ^\uparrow .

Then one may ask how it is possible to get a large direct decay probability for the collective state out of small individual decay rates Γ_μ ? This is due to a coherence effect. Indeed, if we excite the collective state $|Coll\rangle$ and we want to compute its decay probability, in terms of the individual probability Γ_μ , using $|Coll\rangle = c_{Coll, \mu} |\mu\rangle$, we get

$$\Gamma^\uparrow \propto |\langle k | \Delta \hat{V} | Coll \rangle|^2 = \left| \sum_\mu c_{Coll, \mu} \langle k | \Delta \hat{V} | \mu \rangle \right|^2 \quad (10)$$

This coherent sum can be much bigger than the incoherent analogue to $\sum_\mu |c_{Coll, \mu}|^2 \Gamma_\mu$. This means that it is possible to have a strong direct coupling from the collective state to the continuum built out of a coherent sum of many small decay probability.

In the following, the direct decay of the collective state is included in the individual width Γ_μ of each eigenstates $|\mu\rangle$. To do so, the coherence should be kept and the width should fulfill equation (9). However, we will often neglect the energy dependence of the width Γ_μ in the analytical expressions. We have tested numerically that this energy dependence does not affect the presented conclusions.

2.2.3 Microscopic Description of Ericson Fluctuation in Resonant Phenomena

Let us first recall the standard description [17] of a doorway state which includes only the first level. In such a case, assuming $|\langle Coll | \hat{V}_1 | i_1 \rangle| = v_1$,

and a regular spectrum of states $\{|i_1\rangle\}$ with a level spacing $E_{i_1} - E_{i_1-1} = \Delta E_1$, the overlap matrix becomes (see appendix 2)

$$O_{\mu_1} = \frac{v_1^2}{(v_1^2 + v_1^4 \pi^2 / \Delta E_1^2) + (E_{\mu_1} - E_{Coll})^2} \quad (11)$$

which is the standard Lorentzian shape with a typical width $\Gamma_{Coll}^2 = 4(v_1^2 + v_1^4 \pi^2 / \Delta E_1^2)$. In the limit of a continuous spectrum and of $\Delta E_1 \rightarrow 0$ assuming $v_1^2 / \Delta E_1 = cte$, one gets $\Gamma_{Coll} = 2\pi v_1^2 / \Delta E_1$ which is equivalent to the standard Fermi-golden rule.

In this derivation no fluctuation in the strength function is introduced. This can be traced back to the simplifying assumptions about the density of states $|i\rangle$ and the constant interaction matrix elements $\langle Coll | \hat{V}_1 | i_1 \rangle$. Let us, for example, introduce a fluctuating part of the residual interaction matrix element defined by $|\langle Coll | \hat{V}_1 + \delta \hat{V}_1 | i_1 \rangle|^2 = \bar{v}_1^2 (1 + \delta v_{i_1})$. Then the overlap matrix can also be splitted into an averaged and a fluctuating part $O_{\mu_1} = \bar{O}_{\mu_1} + \delta O_{\mu_1}$, the averaged being identical to the constant interaction case (see equation (11) and Appendix 3). Introducing diagonal correlation of the fluctuating matrix element $\delta v_{i_1} \delta v_{j_1} = c_{i_1} \delta_{i_1 j_1}$, we get

$$\delta O_{\mu_1} \delta O_{\nu_1} \simeq \delta_{\mu_1 \nu_1} \delta O_{\mu_1}^2 \quad (12)$$

where the expression of $\delta O_{\mu_1}^2$ is given in appendix 3. Therefore, one get back the conditions needed to observe Ericson fluctuations.

We have investigated this effect using a numerical diagonalization of a non-fluctuating Hamiltonian \hat{H}_1 (top part of fig. (2)) and of a fluctuating one (bottom part of fig. (2)). To describe the finite lifetime of the states $|\mu_1\rangle$, an imaginary part $-i \Gamma_{\mu_1}/2$ has been added to each energy E_{μ_1} . On figure (2), we can see that a fluctuating residual interaction produces Ericson fluctuations. The above discussion applies to the second RPA[6, 20, 21, 22, 23, 24] description of collective excitations which takes into account the decay of collective states excited through one body operators, i.e. the coupling of states built from particle-hole (p-h) type of excitations, into more complex configurations containing two-particle two-hole (2p-2h) states. In this framework, one would expect to observe fluctuations of the collective strength related to the characteristic lifetime of the (2p-2h) states.

2.2.4 Fluctuations within a Microscopic description of the lifetime

of the decay channels

In order to get a deeper insight into the fluctuation mechanism, one can introduce the second level of complexity $|i_2\rangle$. The corresponding collective response can be computed by introducing as follows:

- first the state $|Coll\rangle$ is coupled to the $|i_1\rangle$ by the residual interaction $\hat{V}_1 + \delta\hat{V}_1$ leading to eigenstates $|\mu_1\rangle$

$$|\mu_1\rangle = c_{\mu_1, Coll}^1 |Coll\rangle + \sum_{i_1} c_{\mu_1, i_1}^1 |i_1\rangle \quad (13)$$

- Then, the states $|\mu_1\rangle$ are coupled through the residual interaction \hat{V}_2 with the states $|i_2\rangle$ in order to built more complex eigenstates $|\mu_2\rangle$

$$|\mu_2\rangle = c_{\mu_2, \mu_1}^2 |\mu_1\rangle + \sum_{i_2} c_{\mu_1, i_2}^2 |i_2\rangle \quad (14)$$

Therefore we get

$$|\mu_2\rangle = c_{\mu_2, \mu_1}^2 c_{\mu_1, Coll}^1 |Coll\rangle + c_{\mu_2, \mu_1}^2 \sum_{i_1} c_{\mu_1, i_1}^1 |i_1\rangle + \sum_{i_2} c_{\mu_2, i_2}^2 |i_2\rangle \quad (15)$$

If we assume that the states $|\mu_1\rangle$ are independently coupled to different ensembles of states $|i_2\rangle$ through a constant interaction v_2 , then, each $|\mu_1\rangle$ will act as a doorway state toward its own decay channels and so will be spread over a typical width $\Gamma_{\mu_1}^2 = 4(v_2^2 + v_2^4 \pi^2 / \Delta E_2^2)$ where ΔE_2 is nothing but the level spacing at the states $|i_2\rangle$ coupled to $|\mu_1\rangle$. With this formalism it is clear that the overlap matrix presents in fact two scales

$$O_{\mu_2} = O_{\mu_1}^1 O_{\mu_2, \mu_1}^2 \quad (16)$$

the first one $O_{\mu_1}^1$ associated with the first level of complexity μ_1 , and a second one O_{μ_2, μ_1}^2 associated with the spreading of μ_1 over μ_2 . Within the presented approximation the strength function reads

$$S(E) = \frac{1}{2\pi} \sum_{\mu_1} (\bar{O}_{\mu_1} + \delta O_{\mu_1}) v_2^2 \sum_{\mu_2} \frac{\Gamma_{\mu_2}}{\left((E_{\mu_2} - E_{\mu_1})^2 + \Gamma_{\mu_1}^2/4\right)^2 \left((E - E_{\mu_2})^2 + \Gamma_{\mu_2}^2/4\right)} \quad (17)$$

where we have introduced a width Γ_{μ_2} for the states μ_2 . Describing \bar{O}_{μ_1} as in eq. (11) and the δO_{μ_1} as fluctuation, we recover to the case of Ericson fluctuations as discussed in the previous chapter.

2.3 Multiscale Fluctuations and the Doorway Hierarchy

In the previous section, we have assumed that the doorway state was directly coupled to a large ensemble of states with an average intrinsic width. In such a case, fluctuations with a typical scale, related to this width, are observed in the response spectrum. We have also considered the possibility that the damping might be due to a coupling to more complex states. However, we have not yet investigated the possibility that this second level of complexity might also be fluctuating.

2.3.1 Multiscale Ericson Fluctuation

We expect in general fluctuations at this second step of the decay, then the strength reads

$$S(E) = \frac{1}{2\pi} \sum_{\mu_1} (\bar{O}_{\mu_1}^1 + \delta O_{\mu_1}^1) \sum_{\mu} (\bar{O}_{\mu_2, \mu_1}^2 + \delta O_{\mu_2, \mu_1}^2) \frac{\Gamma_{\mu_2}}{(E - E_{\mu_2})^2 + \Gamma_{\mu_2}^2/4} \quad (18)$$

Using a lorentzian shape both for $\bar{O}_{\mu_1}^1$ with a width Γ_{Coll} and for \bar{O}_{μ_2, μ_1}^2 with a width Γ_{μ_1} , we can see that two scales Γ_{μ_1} and Γ_{μ_2} are present in the fluctuations of the strength function on top of the spreading of the collective state Γ_{Coll} . This is illustrated in figure 3 (top-right) where we have plotted a strength computed with two scales for Ericson fluctuations.

2.3.2 Fluctuations of a fragmented strength

Another interesting situation is the case of Landau spreading. Indeed, at the RPA level of description[16, 25], the strength often appears splitted in several components $|Coll_n\rangle$. In such a case, at least three scales might be identified. The Landau spreading can be directly observe as the fragmentation of the strength while the width of each fragment can be identified with the lifetime of each individual collective state. Moreover, as discussed above the strength presents fluctuations on top of each resonant line-shape which are characteristic of the various decay channels widths. Indeed, if we assume that the different fragments are decaying toward independent complex states, the strength is simply the sum of the various components associated with a

collective excitation $|Coll_n\rangle$

$$S(E) = \sum_n |\langle Coll | Coll_n \rangle|^2 \left(\sum_\mu \frac{|\langle \mu | Coll_n \rangle|^2}{(E - E_\mu) + i\Gamma_\mu/2} \right) \quad (19)$$

Such a fragmented strength is illustrated in figure 3 (top-right).

3 Observation of fluctuations at different scales

The observation of the collective state width Γ_{Coll} , the largest scale in the strength, is rather straightforward using standard techniques such as fits, variance estimation or even autocorrelation calculations. The extraction of characteristic widths from a spectrum presenting many different scales of fluctuations is a more complicated problem. In this chapter, we discuss several methods and propose new approaches for extracting signals of multiscale fluctuations.

3.1 Standard method and its extension.

We have seen in the previous chapter that the autocorrelation function is a useful tool when fluctuations and fragmentation of the response are involved (see figure (1)). On the one hand, when the autocorrelation techniques is used directly on the strength distribution, the obtained signal can be related the total width of the collective mode. A standard technique to extract the fluctuation properties of a fine structure on top of some smooth "background" is to subtract an averaged distribution[18] from the signal. However, the obtained result appears to be dependent on the method and parameters (such as the smoothing interval) used to define this average strength. In order to overcome this problem, we have performed the autocorrelation analysis on the derivative of the strength distribution. The derivative is very sensitive to the fluctuations over the smaller scale present in the spectrum since they are associated with very rapid variations. In this case, the autocorrelation function gives half of the width of the smallest fluctuations scale.

These two types of autocorrelation analyses, on the strength and on its derivative, are illustrated in figures (3) for two cases: a multiscale (two scales) Ericson fluctuation (left of fig. (3)) and a fragmented strength with fine structure (right of fig. (3)).

In both cases (multiscale Ericson or fragmented), the largest and the smallest scales in the fluctuations can be extracted. Considering now the fragmented strength case, an intermediate scale can also be observed. However, when Ericson fluctuations occurs at many different scales, the auto-correlations techniques applied to the strength and its derivative only gives access to the largest and the smallest scales of the fluctuations and intermediates scales could not be obtained.

During the past years, different alternative techniques have been proposed to extract properties of fluctuations in the decay of collective motions [26, 27, 28, 29, 30]. In particular, in a second RPA picture, where only two-body correlations are retained, a possible fractal nature of the fluctuations has been discussed [27, 28, 29, 30]. When higher correlations are considered, we have shown that a hierarchy of well-separated scales may also be present. In the next section, we will propose a novel technique, that may help to get either the well separated scales or the fractal nature of nuclear decay.

3.2 Entropy index for multiscale fluctuation

Recently, a method based on the definition of an entropy index has been proposed by Hwa[31] in order to extract scaling behavior in fluctuating signal. The entropy index appears as a good indicator of the existence of different scales in a strongly fluctuating spectrum. We have adapted this method to the nuclear response case.

The total energy interval $\Delta E = E_{MAX} - E_{MIN}$, is first divided into n bins of resolution δE ($n = \Delta E/\delta E$). The signal in each bin is analyzed according to a simple overlap with a step function changing from -1 to $+1$ in the middle of the considered bin⁴. Therefore, at every scale δE , in each of the bin j , a coefficient $D_j(\delta E)$ is defined as

$$D_j(\delta E) = \int_{E_{MIN}+(j-1)\delta E}^{E_{MIN}+j\delta E} dE S(E) \text{sign}(E - (j - 1/2)\delta E) \quad (20)$$

The coefficients D_j can be considered as a coarse grained derivative of S . In order to focus on the global properties of the fluctuations at a given size δE , an entropy factor $K(\delta E)$ can be defined as

$$K(\delta E) = \frac{1}{n} \sum_{j=1,n} W_j(\delta E) \log W_j(\delta E) \quad (21)$$

⁴This method can be viewed as a wavelet analysis.

where the coefficient $W_j(\delta E) = D_j / \langle D_j \rangle$ are nothing but the coefficients D_j normalized to their averaged value ($\langle D_j \rangle = 1/n \sum_{j=1,n} D_j$).

In the reference[31], it is shown that a linear decrease of $K(\delta E)$ as a function of $\log(\delta E)$ characterizes the existence of fluctuations at all scales. When fluctuations with specific scales are considered, a different behavior is expected. Having in mind that the coefficients $W_j(\delta E)$ might be interpreted as a normalized coarse-grained derivative, $K(\delta E)$ should remain almost constant between two typical scales since in-between these scales, the coarse grained derivative of S do not vary much. However, when going from one scale to another, the derivative loses part of its structure and one is expecting an entropy variation. This situation is illustrated in figure (4) for three different cases, where respectively fluctuations over one, two and three scales have been introduced in the strength. Note that, in order to avoid problems due to the limited number of bins for large bin size, we have considered a function defined as a repetition of the strength function instead of the strength itself. In practice, we used 31 repetitions of the strength. The entropy index is then applied on the new function inside a large energy interval, enabling to have a large number of bins in the energy region under interest ($\delta E \leq \Delta E$). From fig. (4), one can observe that the evolution of $K(\delta E)$ indicates the presence of respectively one, two and three scales by a change in its curvature. In order to emphasize this evolution, we have plotted in figure (5) a numerical estimation of the second derivative $K''(\delta E)$ of the entropy index. The presence of one, two or three scales is thus signed by the presence of respectively one, two and three minima in these second derivative.

In all the presented cases, $K(\delta E)$ is a good indicator of the different characteristic scales in the nuclear response. Using the positions of the curvature variations, the various scales can be roughly estimated.

In this section, the entropy index method was applied to model where scales are well separated. This situation may appear at rather low excitation energy where few degrees of freedom are coupled to collective states[32]. When excitation energy increases, the number of internal and external degrees of freedom involved in the coupling becomes very large[33] and a statistical treatment is required [17, 18, 34]. In this case, many different scales are expected and can be uncovered through the entropy index method. This method can also be used in more complex situations where mixing of the different scales is expected. It can even sign self-similar fluctuations [26, 27, 29]. It should be noticed that the smallest scales expected in the strength are re-

lated to the lifetime of the compound nucleus states which are less than the eV. However, experimental resolutions have not yet reached this degree of accuracy [35, 36]. Elaborated techniques based on statistical assumptions have been applied in order to extract the properties of invisible fine structures[18]. The entropy index method can be viewed as a model independent way to extract information about fluctuations. Only scales larger than the experimental resolution can be accessed but no assumptions on the statistical properties of the studied spectrum are needed.

4 Critical discussion of the Doorway hierarchy picture

In order to observe multiscale fluctuations, we have assumed a particular hierarchy in the Hamiltonian following the general scenario of a gradual complexification of a collective motion until it reaches the compound nucleus chaotic states. It is in fact the implicit assumption of many simulations such as the extended mean-field approach which takes into account the two-body collisions as a damping mechanism[32, 37, 38, 39, 40]. In these cases, a hierarchy of couplings is implicitly assumed, the main ansatz being that cutting this hierarchy at any level leads to an approximate strength which is only slightly modified when the next level is introduced. From a quantal point of view, this means that an ensemble of states ordered by increasing complexity, such as states with increasing number of quasi-particle excitations, can be defined and that each level of complexity can be considered as a perturbation on top of the previous level. However, there is no *a priori* reason that such a robust hierarchy exists. Indeed, in quantum mechanics, a modification of few matrix elements is often sufficient to introduce an important rearrangement of the whole spectrum. Therefore, even if the hierarchy of doorway seems valid for several decay steps, it is always possible that the introduction of the next level of complexity deeply transforms the overall picture. For example, the coupling of overlapping resonances through their common decay channels has been already discussed in connection with nuclear relaxation (see for instance [13, 15]). Specific coherent effects, like the Dicke superradiance well known in quantum optic[41], could also be present in nuclear spectra and could suppress Ericson fluctuations[14]. Such coherence effects can also affect the nuclear response when a hierarchy of intrinsic degrees of freedom

is considered.

Let us consider a single collective doorway state $|Coll\rangle$ damped through a residual interaction \hat{V}_1 and \hat{V}_2 assuming that $\langle Coll|\hat{V}_1|i_1\rangle = v_1$, $\langle i_1|\hat{V}_2|i_2\rangle = v_2$ and that the second level of complexity $|i_2\rangle$ is characterized by a level spacing ΔE_2 . Then we can take the limit $v_2^2 \rightarrow 0$ and $\Delta E_2 \rightarrow 0$ keeping $\Gamma_1 = 2\pi v_2^2/\Delta E_2$ constant. In the case of a regular ensemble of states i_1 with a spacing ΔE_1 , we can write the overlap matrix (see Appendix 4)

$$O_{\mu_2} = |c_{Coll}^{\mu_2}|^2 = \frac{v_2^2 v_1^2}{\frac{\Gamma_1^2}{4} (E_{\mu_2} - E_{Coll})^2 + \frac{\Delta E_1^2}{\pi^2} \left((E_{\mu_2} - E_{Coll}) \tan\left(\pi \frac{E_{\mu_2}}{\Delta E_1}\right) - \frac{\Gamma_{Coll}}{2} \right)^2} \quad (22)$$

where we have introduced $\Gamma_{Coll} = 2\pi v_1^2/\Delta E_1$. When $\Gamma_1 \gg \Delta E_1$. In such a case, the overlap O_{μ_2} has a typical scale of fluctuation not related to Γ_1 but to ΔE_1 since the denominator goes to infinity each $E_{\mu_2} = (n + 1/2)\Delta E_1$ in eq. (22).

Again we have tested that all type of fluctuations of the residual Hamiltonian as illustrated in figure (6). This figure presents strong oscillations with a width related to ΔE_1 as shown by the autocorrelation function. Therefore, the occurrence of interference like pattern characteristic of the level spacing ΔE_1 should be considered as generic. We would also like to mention that a strong coupling between the first and second decay channel induces a narrowing of the apparent width of the collective states. This effect is analogous to the motional narrowing described in [42].

This illustration stresses the fact that in the case of a coupling of several states through their decay channels one should expect interference pattern due to a feedback of the decay channels on the properties and the coupling of the doorway states. Then, the fluctuation pattern can reflect the typical level spacing and not the typical width.

5 Conclusion

In conclusion, we have studied the role of several steps in the decay of doorway states. We have shown that, in some cases, in particular when the decay channels of each states encountered at each level of the decay cascade can be considered as independent, one should expect to observe several scales in the fluctuation pattern of the response function. In such a case, the various fluctuations are characteristic of the typical lifetime at each level of

complexity encountered in the path from the doorway toward the chaotic compound nucleus states. We have shown that this multiscale Ericson fluctuations can be partially revealed by applying the autocorrelation technique on the strength. However, standard methods which involve different smoothing procedures seem to bias the obtained results. In order to overcome this problem, we have proposed two new techniques: the first one consists in computing the autocorrelation function on the derivative of the strength distribution in order to extract the smallest scale in the fluctuations, the second one, called Entropy index, gives information on how many scales does exist in the fluctuation pattern. Therefore, high resolution experiments are indeed very interesting tools to study the damping mechanism of collective states.

Finally, the analytical results as well as the numerical simulations of the last chapter illustrate that a hierarchy of scales in the fluctuation, directly related to the lifetime of each level of complexity, is not the generic case and that often the fluctuations can be related to other physical quantities. In particular we have investigated the effects of decay channels interacting via the states encountered at a higher level of complexity. In this case, interferences between different decay channels are found and the induced fluctuations are not related to the decay width of the considered states but to their spacing. In such a case, experiment with a very high resolution will present fluctuations which are characteristic of the level spacing and not of the width as in Ericson fluctuations.

Caution should thus be used when interpreting the observed width of the fluctuations. Indeed, the interferences, due to an interaction of some states sharing the same decay channels, may produce narrow fluctuations which may mimic long lived systems. This is an important finding which can modify the interpretation of Experimental data.

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APPENDIX 1: Ericson Fluctuations in doorway processes

Let us consider the spectral response

$$R(E) = \sum_{\mu} \frac{|\langle \mu | Coll \rangle|^2}{(E - E_{\mu}) + i\Gamma_{\mu}/2} \quad (23)$$

When we introduce an averaged overlap and a fluctuating one, $|\langle \mu | Coll \rangle|^2 = \bar{O}(E_\mu) + \delta O_\mu$, the response can also be split into two components. On the one hand, it contains an averaged response function $\bar{R}(E) = \int_{\delta E} dE' R(E')$ which can be written as for $E > 0$

$$\bar{R}(E) \simeq \frac{1}{\delta E} \int_{\delta E} dE' \rho(E') \frac{\bar{O}(E')}{(E - E') + i\bar{\Gamma}/2} \simeq \bar{\rho} \bar{O}(E) \quad (24)$$

where we have introduced the density of states $\rho(E) = \sum_\mu \delta(E - E_\mu)$, the average density of states $\bar{\rho} = \int_{\delta E} dE' \rho(E') / \delta E = 1/\Delta E$ and an averaged width $\bar{\Gamma}$ (the integration interval δE being taken much larger than $\bar{\Gamma}$). On the other hand remains a fluctuating term

$$\delta R(E) = \sum_\mu \frac{\delta O_\mu}{(E - E_\mu) + i\Gamma_\mu/2} \quad (25)$$

These fluctuations still contain information about the averaged width $\bar{\Gamma}$ as it was demonstrated by Ericson[12]. Indeed, the autocorrelation function of this noise is given by

$$C_R(E) = \frac{1}{\delta E} \int_{E_0}^{E_0+\delta E} dE' \delta R^*(E') \delta R(E' + E) \quad (26)$$

$$C_R(E) = \sum_{\mu\nu} \frac{1}{\delta E} \int_{E_0}^{E_0+\delta E} dE' \frac{\delta O_\mu}{E' - \mathcal{E}_\mu^*} \frac{\delta O_\nu}{E' + E - \mathcal{E}_\nu} \quad (27)$$

where we have introduced complex energy notations $\mathcal{E}_\mu = E_\mu - i\Gamma_\mu/2$. If we assume that $\delta E \gg \bar{\Gamma}$, we can extend the integration to infinity, so that, taking advantage of the relation $(x - y)^{-1} (x - z)^{-1} = (y - z)^{-1} ((x - y)^{-1} - (x - z)^{-1})$, it is possible to perform a Cauchy integral leading to

$$C_R(E) = \sum_{\mu \in \Delta E} \frac{2i\pi}{\delta E} \sum_\nu \frac{\delta O_\mu \delta O_\nu}{\mathcal{E}_\nu - E - \mathcal{E}_\mu^*} + \mu \longleftrightarrow \nu \quad (28)$$

If we assume random correlations between the overlap fluctuation

$$\delta O_\mu \delta O_\nu = \delta O_\mu^2 \delta_{\mu\nu} \quad (29)$$

in average, we get the correlation

$$C_R(E) = \frac{4\pi}{\Delta E} \frac{\overline{\delta O^2}}{E^2 + \bar{\Gamma}^2} \quad (30)$$

A similar equation holds for the strength function associated to equation (2)

$$S(E) = -\frac{1}{\pi} \text{Im}(R(E)) = \sum_{\mu} \frac{\frac{\Gamma_{\mu}}{2\pi} |\langle \mu | \text{Coll} \rangle|^2}{(E - E_{\mu})^2 + \Gamma_{\mu}^2/4} \quad (31)$$

Indeed, we can define the autocorrelation function of the strength distribution as

$$C(E) = \frac{1}{\delta E} \int_{E_0}^{E_0+\delta E} dE' \delta S(E') \delta S(E' + E) \quad (32)$$

Noting that $S(E) = -1/(2i\pi) (R(E) - R^*(E))$ we have

$$C(E) = \frac{2}{\pi \Delta E} \frac{\overline{\delta O^2}}{E^2 + \overline{\Gamma}^2} \quad (33)$$

This relation shows that, even in the case of strongly overlapping states, one can still extract information about the typical width by looking at the fluctuating part of the considered autocorrelation spectrum as it was demonstrated in [12].

APPENDIX 2: Simple Doorway Picture of a Damping Mechanism

The diagonalization of the Hamiltonian $\hat{H}_1 = \hat{H}_0 + \hat{V}_1$ produces eigenstates $|\mu_1\rangle$ associated with the eigenenergies E_{μ_1} which fulfill the following dispersion relation

$$E_{\mu_1} - E_{\text{Coll}} = f_1(E_{\mu_1}) v_1^2 \quad (34)$$

where we have considered a constant interaction $v_1^2 = |\langle \text{Coll} | \hat{V} | i_1 \rangle|^2$. The overlap matrix $O_{\mu_1} = |\langle \mu_1 | \text{Coll} \rangle|^2$ reads

$$O_{\mu_1} = \frac{1}{1 + f_2(E_{\mu_1}) v_1^2} \quad (35)$$

In the previous equations, we have introduced the functions f_n

$$f_n(E) = \sum_{i_1} \frac{1}{(E - E_{i_1})^n} \quad (36)$$

which are related by a recurrence relation

$$f_{n+1}(E) = -n \frac{\partial f_n(E)}{\partial E} \quad (37)$$

In order to get the usual Lorentzian shape, we assume a regular spectrum of energy E_{i_1} : $E_{i_1} - E_{i_1-1} = \Delta E_1$ with i_1 running over both positive and negative integer values. Then, using the relation $\sum_i 1/(x - i) = \pi \cot(\pi x)$, the dispersion relation becomes

$$E_{\mu_1} - E_{Coll} = f_1(E_{\mu_1}) = \frac{v_1^2 \pi}{\Delta E_1} \cot\left(\pi \frac{(E_{\mu_1} - E_{Coll})}{\Delta E_1}\right) \quad (38)$$

On the other hand, using the relation (37) and taking advantage of the dispersion relation (38), $f_2(E_{\mu_1})$ can be recast as

$$f_2(E_{\mu_1}) = \left(\frac{\pi^2}{\Delta E_1^2} + \frac{(E_{\mu_1} - E_{Coll})^2}{v_1^4} \right) \quad (39)$$

so that the overlap matrix simply reads

$$O_{\mu_1} = \frac{v_1^2}{(v_1^2 + v_1^4 \pi^2 / \Delta E_1^2) + (E_{\mu_1} - E_{Coll})^2} \quad (40)$$

which is the standard Lorentzian shape with a typical width $\Gamma_{Coll}^2 = 4(v_1^2 + v_1^4 \pi^2 / \Delta E_1^2)$.

APPENDIX 3: Microscopic Description of Ericson Fluctuation in Resonant Phenomena

Following Appendix 2, let us, for example, introduce a small fluctuating matrix $\delta \hat{V}_1$. Then we define a fluctuating part for the f_n functions

$$\delta f_n(E) = \sum_{i_1} \frac{\delta v_{i_1}}{(E - E_{i_1})^n} \quad (41)$$

where we have used the notation $|\langle Coll | \hat{V}_1 + \delta \hat{V}_1 | i_1 \rangle|^2 = \overline{v_1^2}(1 + \delta v_{i_1})$. The new dispersion relation reads

$$E_{\mu_1} - E_{Coll} = \overline{v_1^2}(f_1(E_{\mu_1}) + \delta f_1(E_{\mu_1})) \quad (42)$$

while the overlap becomes

$$O_{\mu_1} = \frac{1}{1 + \overline{v_1^2}(f_2(E_{\mu_1}) + \delta f_2(E_{\mu_1}))} \quad (43)$$

In the case of a regular spectrum of states $|i_1\rangle$, using the relations derived above, the dispersion relation becomes

$$E_{\mu_1} - E_{Coll} = \overline{v_1^2} \left(\frac{\pi}{\Delta E_1} \cot\left(\pi \frac{(E_{\mu_1} - E_{Coll})}{\Delta E_1}\right) + \delta f_1(E_{\mu_1}) \right) \quad (44)$$

so that, we may use this relation in order to compute $f_2(E_{\mu_1})$

$$f_2(E_{\mu_1}) = \left(\frac{\pi^2}{\Delta E_1^2} + \bar{v}_1^{-4} \left(E_{\mu_1} - E_{Coll} - \bar{v}_1^{-2} \delta f_1(E_{\mu_1}) \right)^2 \right) \quad (45)$$

The latter equation can be introduced in the overlap matrix leading the perturbative result

$$O_{\mu_1} = \bar{O}_{\mu_1} + \delta O_{\mu_1} \quad (46)$$

with an averaged value, \bar{O}_{μ_1} , identical to the one derived in appendix 2 and with a fluctuating part given by

$$\delta O_{\mu_1} = \bar{O}_{\mu_1}^2 \left(2(E_{\mu_1} - E_{Coll}) \delta f_1(E_{\mu_1}) - \bar{v}_1^{-2} \delta f_2(E_{\mu_1}) \right) \quad (47)$$

We finally get :

$$\delta O_{\mu_1} = \bar{O}_{\mu_1}^2 \sum_{i_1} \delta v_{i_1} \delta o_{\mu_1}(E_{i_1}) \quad (48)$$

with

$$\delta o_{\mu_1}(E) = \frac{1}{(E_{\mu_1} - E)} \left(\frac{2(E_{\mu_1} - E_{Coll})}{\bar{v}_1^2} - \frac{1}{E_{\mu_1} - E} \right) \quad (49)$$

This is a fluctuating correction to the averaged overlap which looks like the one needed in order to observe Ericson-like fluctuations. Indeed, the correlation $\overline{\delta O_{\mu_1} \delta O_{\nu_1}}$ reads as a function of the correlation of the fluctuating matrix element $\overline{\delta v_{i_1} \delta v_{j_1}} = c_{i_1 j_1}$

$$\overline{\delta O_{\mu_1} \delta O_{\nu_1}} = \bar{O}_{\mu_1}^2 \bar{O}_{\nu_1}^2 \sum_{i_1 j_1} c_{i_1 j_1} \delta o_{\mu_1}(E_{i_1}) \delta o_{\nu_1}(E_{j_1}) \quad (50)$$

If we now assume uncorrelated fluctuations for the matrix elements $c_{i_1 j_1} = c_{i_1} \delta_{i_1 j_1}$ we get

$$\overline{\delta O_{\mu_1} \delta O_{\nu_1}} = \bar{O}_{\mu_1}^2 \bar{O}_{\nu_1}^2 \sum_{i_1} c_{i_1} \delta o_{\mu_1}(E_{i_1}) \delta o_{\nu_1}(E_{i_1}) \quad (51)$$

because of the dispersion relation, the energy E_{μ_1} is always very close from one particular state $|i_1\rangle$ which we note $|i_1^{\mu_1}\rangle$ associated to an unperturbed energy $E_{i_1^{\mu_1}}$. Then, in δo_{μ_1} , mainly this state $i_1^{\mu_1}$ is participating to the sum over i_1 so that $\delta O_{\mu_1} \approx \bar{O}_{\mu_1}^2 \delta v_{i_1^{\mu_1}} \bar{o}_{\mu_1}(E_{i_1^{\mu_1}})$ and the correlation reads

$$\overline{\delta O_{\mu_1} \delta O_{\nu_1}} \approx \delta_{\mu_1 \nu_1} \bar{O}_{\mu_1}^4 c_{i_1^{\mu_1}} \delta o_{\mu_1}^2(E_{i_1^{\mu_1}}) = \delta_{\mu_1 \nu_1} \delta O_{\mu_1}^2 \quad (52)$$

Therefore, one get back the correlations needed to observe Ericson fluctuations. However, it should be noticed that compared with the simple constant correlation assumed in eq. (29) one may have here a smooth energy dependence of $\delta O_{\mu_1}^2$. This however do affect the conclusions reached about the width of the autocorrelation function.

APPENDIX 4: Interferences in the Damping of a Resonance

We can finally consider the related case of a single doorway collective state $|Coll\rangle$ damped through a residual interaction \hat{V}_1 toward a first level of complexity described by the states $|i_1\rangle$ which are then coupled to many states $|i_2\rangle$ by a second part of the residual interaction \hat{V}_2 . Then, the diagonalization of $\hat{H} = \hat{H}_0 + \hat{V}_1 + \hat{V}_2$ (where $|Coll\rangle$, $|i_1\rangle$ and $|i_2\rangle$ are eigenstates of \hat{H}_0) produces eigenstates

$$|\mu_2\rangle = c_{\mu_2,Coll} |Coll\rangle + \sum_{i_1} c_{\mu_2,i_1} |i_1\rangle + \sum_{i_2} c_{\mu_2,i_2} |i_2\rangle \quad (53)$$

The Schrödinger equation leads to

$$c_{\mu_2,Coll} (E_{\mu_2} - E_{Coll}) = \sum_{i_1} c_{\mu_2,i_1} \langle Coll | \hat{V}_1 | i_1 \rangle = v_1 a_{\mu_2}^1 \quad (54)$$

$$c_{\mu_2,i_1} (E_{\mu_2} - E_{i_1}) = c_{\mu_2,Coll} \langle i_1 | \hat{V}_1 | Coll \rangle + \sum_{i_2} c_{\mu_2,i_2} \langle i_1 | \hat{V}_2 | i_2 \rangle = v_1 c_{\mu_2,Coll} + v_2 a_{\mu_2}^2 \quad (55)$$

$$c_{\mu_2,i_2} (E_{\mu_2} - E_{i_2}) = \sum_{i_1} c_{\mu_2,i_1} \langle i_1 | \hat{V}_2 | i_2 \rangle^* = v_2 a_{\mu_2}^1 \quad (56)$$

where we have assumed that $\langle Coll | \hat{V}_1 | i_1 \rangle = v_1$, $\langle i_1 | \hat{V}_2 | i_2 \rangle = v_2$ and where we have introduced $a_{\mu_2}^1 = \sum_{i_1} c_{\mu_2,i_1}$, $a_{\mu_2}^2 = \sum_{i_2} c_{\mu_2,i_2}$. The first equation gives

$$c_{\mu_2,Coll} = \frac{v_1 a_{\mu_2}^1}{E_{\mu_2} - E_{Coll}} \quad (57)$$

so that the two other relations reads

$$c_{\mu_2,i_1} = \frac{1}{E_{\mu_2} - E_{i_1}} \left(\frac{v_1^2 a_{\mu_2}^1}{E_{\mu_2} - E_{Coll}} + v_2 a_{\mu_2}^2 \right) \quad (58)$$

$$c_{\mu_2,i_2} = \frac{v_2 a_{\mu_2}^1}{E_{\mu_2} - E_{i_2}} \quad (59)$$

Therefore, the following relations between a_1 and a_2 can be expressed as

$$a_{\mu_2}^2 = \sum_{i_2} \frac{v_2 a_{\mu_2}^1}{E_{\mu_2} - E_{i_2}} \equiv v_2 a_{\mu_2}^1 f_1(E_{\mu_2}) \quad (60)$$

$$a_{\mu_2}^1 = \sum_{i_1} \frac{v_1^2}{E_{\mu_2} - E_{i_1}} \frac{a_{\mu_2}^1}{E_{\mu_2} - E_{Coll}} + \frac{v_2 a_{\mu_2}^2}{E_{\mu_2} - E_{i_1}} \equiv \left(\frac{v_1^2 a_{\mu_2}^1}{E_{\mu_2} - E_{Coll}} + v_2 a_{\mu_2}^2 \right) F_1(E_{\mu_2}) \quad (61)$$

where we have introduced the functions f_n and F_n

$$F_n(E_{\mu_2}) = \sum_{i_1} \frac{1}{(E_{\mu_2} - E_{i_1})^n} \quad (62)$$

$$f_n(E_{\mu_2}) = \sum_{i_2} \frac{1}{(E_{\mu_2} - E_{i_2})^n} \quad (63)$$

Then the eigenenergy E_{μ_2} fulfills the following dispersion relation

$$1 = \left(\frac{v_1^2}{E_{\mu_2} - E_{Coll}} + v_2^2 f_1(E_{\mu_2}) \right) F_1(E_{\mu_2}) \quad (64)$$

Moreover, the a coefficients can be found using the normalization condition

$$1 = \frac{v_1^2 a_{\mu_2}^{1^2}}{(E_{\mu_2} - E_{Coll})^2} + v_2^2 a_{\mu_2}^{1^2} f_2(E_{\mu_2}) + \left(\frac{v_1^2 a_{\mu_2}^1}{E_{\mu_2} - E_{Coll}} + v_2 a_{\mu_2}^2 \right)^2 F_2(E_{\mu_2}) \quad (65)$$

which leads to a condition on a , through the use of the relation (60), in order to remove $a_{\mu_2}^2$,

$$1 = \frac{v_1^2 a_{\mu_2}^{1^2}}{(E_{\mu_2} - E_{Coll})^2} + v_2^2 a_{\mu_2}^{1^2} f_2(E_{\mu_2}) + \left(\frac{v_1^2 a_{\mu_2}^1}{E_{\mu_2} - E_{Coll}} + v_2^2 a_{\mu_2}^1 f_1(E_{\mu_2}) \right)^2 F_2(E_{\mu_2}) \quad (66)$$

allowing to extract the coefficient a_1

$$a_{\mu_2}^{1^2} = \left(\frac{v_1^2}{(E_{\mu_2} - E_{Coll})^2} + v_2^2 f_2(E_{\mu_2}) + \left(\frac{v_1^2}{E_{\mu_2} - E_{Coll}} + v_2^2 f_1(E_{\mu_2}) \right)^2 F_2(E_{\mu_2}) \right)^{-1} \quad (67)$$

Then the overlap matrix, $O_{\mu_2} = |c_{\mu_2, Coll}|^2$, reads

$$O_{\mu_2} = \frac{v_1^2}{v_1^2 + v_2^2 f_2(E_{\mu_2}) (E_{\mu_2} - E_{Coll})^2 + (v_1^2 + v_2^2 f_1(E_{\mu_2}) (E_{\mu_2} - E_{Coll}))^2 F_2(E_{\mu_2})} \quad (68)$$

Let us now assume as usual that the second level of complexity $|i_2\rangle$ is characterized by a level spacing ΔE_2 , then we can write for f_1

$$f_1(E) = \frac{\pi}{\Delta E_2} \cot\left(\pi \frac{E}{\Delta E_2}\right) \quad (69)$$

Using the relation (37), the corresponding relation for f_2 can be obtained

$$f_2(E) = \frac{\pi^2}{\Delta E_2^2} \left(1 + \cot^2\left(\pi \frac{E}{\Delta E_2}\right)\right) = \left(\frac{\pi^2}{\Delta E_2^2} + f_1^2(E)\right) \quad (70)$$

We get the dispersion relation

$$f_1(E_{\mu_2}) = \frac{F_1^{-1}(E_{\mu_2})}{v_2^2} - \frac{v_1^2}{(E_{\mu_2} - E_{Coll}) v_2^2} \quad (71)$$

then we get for the coupling coefficient $c_{i_1}^{\mu_2}$

$$O_{\mu_2} = |c_{\mu_2, Coll}|^2 = \frac{v_2^2 v_1^2}{v_2^2 v_1^2 + \frac{\Gamma_2^2}{4} (E_{\mu_2} - E_{Coll})^2 + \left(\frac{(E_{\mu_2} - E_{Coll})}{F_1(E_{\mu_2})} - v_1^2\right)^2 + \frac{(E_{\mu_2} - E_{Coll})^2 v_2^2 F_2(E_{\mu_2})}{F_1^2(E_{\mu_2})}} \quad (72)$$

where we introduce $\Gamma_1 = 2\pi v_2^2/\Delta E_2$. If we assume that the interaction $v_2^2 \rightarrow 0$ we can approximate the overlap by

$$O_{\mu_2} = |c_{Coll}^{\mu_2}|^2 = \frac{v_2^2 v_1^2}{\frac{\Gamma_1^2}{4} (E_{\mu_2} - E_{Coll})^2 + \left(\frac{E_{\mu_2} - E_{Coll}}{F_1(E_{\mu_2})} - v_1^2\right)^2} \quad (73)$$

Let us now consider the case of a regular ensemble of states i_1 with a spacing ΔE_1 , we can write for F_1

$$F_1(E_{\mu_2}) = \frac{\pi}{\Delta E_1} \cot\left(\pi \frac{E_{\mu_2}}{\Delta E_1}\right) \quad (74)$$

and the overlap reads

$$O_{\mu_2} = |c_{Coll}^{\mu_2}|^2 = \frac{v_2^2 v_1^2}{\frac{\Gamma_1^2}{4} (E_{\mu_2} - E_{Coll})^2 + \frac{\Delta E_1^2}{\pi^2} \left((E_{\mu_2} - E_{Coll}) \tan\left(\pi \frac{E_{\mu_2}}{\Delta E_1}\right) - \frac{\Gamma_{Coll}}{2}\right)^2} \quad (75)$$

where we have introduced $\Gamma_{Coll} = 2\pi v_1^2/\Delta E_1$.

References

- [1] G. F. Bertsch, P.F. Bortignon and R. A. Broglia, Rev. Mod. Phys. **55** (1983) 287.
- [2] G.F. Bertsch and R.A. Broglia, *Oscillation in finite quantum systems*, Cambridge (1994).
- [3] Ph. Chomaz and N. Frascaria, Phys Rep. **252** (1995) 275.
- [4] P. A. Moldauer, Phys. Rev. **171** (1968) 1164
- [5] P. Kleinwächter and I. Rotter, Phys. Rev. **C32** (1985) 1742.
- [6] S. Drozdz, S. Nishizaki, J. Speth and J. Wambach, Phys. Rep. **197** (1990) 1.
- [7] Ph. Chomaz, Nucl. Phys. **A569** (1994) 203c.
- [8] Ph. Chomaz, Phys. Lett. **B347** (1995) 1.
- [9] B. Lauritzen, P.F. Bortignon, R.A. Broglia and V.G. Zelevinsky, Phys. Rev. Lett. **74** (1995) 5190.
- [10] V. Zelevinsky, Annu. Rev. Nucl. Part. Sci. **46** (1996) 237.
- [11] J. Winchenbach, K. Pingel, G. Holzmarth, G. Kühner and A. Richter, Nucl. Phys. **A410** (1983) 237.
- [12] T. Ericson, Phys. Rev. Lett. **5** (1960) 430; Ann. Phys. **23** (1963) 390.
- [13] C. Mahaux and H.A. Weidenmüller, *Shell Model Approach to the Nuclear Reactions*, North Holland, Amsterdam (1969).
- [14] V.V. Sokolov and V.G. Zelevinsky, Nucl. Phys. **A504** (1989) 562; Ann. Phys. **216** (1992) 323.
- [15] V.V. Sokolov, I. Rotter, D.V. Savin and M. Müller, Phys. Rev. **C56** (1997) 1031; Phys. Rev. **C56** (1997) 1044.
- [16] P. Ring and P. Schuck, *The nuclear many-body problem*, Springer-Verlag, New-York (1980).

- [17] A. Bohr and B. R. Mottelson, Nuclear Structure, vol. I (Chap. 2).
- [18] G. Kilgus, G. Kühner, S. Müller, A. Richter and W. Knüpfer, Z. Phys. **A326** (1987) 41.
- [19] V.V. Sokolov and V.G. Zelevinsky, Phys. Rev. **C56** (1997) 311.
- [20] H. Suhl and N. R. Werthamer, Phys. Rev. **122** (1961) 359.
- [21] J. Sawicki, Phys. Rev **126** (1962) 2231.
- [22] J. Da Providência, Nucl. Phys. **61** (1965) 87.
- [23] C. Yannouleas *et al*, Nucl. Phys. **A397** (1983) 239.
C. Yannouleas, Phys. Rev. **C35** (1987) 1159.
- [24] C. Yannouleas and S. Yang, Nucl. Phys. **A455** (1986) 40.
- [25] D.J. Rowe, *Nuclear Collective Motion: Models and Theory* , (Methuen, London, 1970).
- [26] S. Drożdż, S. Nishizaki and J. Wambach, Phys. Rev. Lett. 72 (1994) 2839.
- [27] S. Drożdż, S. Nishizaki and J. Wambach and J. Speth, Phys. Rev. Lett. 74 (1995) 1075.
- [28] S. Drożdż, A. Trellakis and J. Wambach, Phys. Rev. Lett. 76 (1996) 4891.
- [29] A. Bershadskii, Phys. Rev. **C59** (1999) 3469.
- [30] H. Aiba and A. Matsuo and , Preprint nucl-th/9904037, to be published in Phys. Rev. C.
- [31] R.C. Hwa, physics/9809041.
- [32] D. Lacroix, Ph. Chomaz and S. Ayik, Phys. Rev. **C58** (1998) 2154.
- [33] N. Frazier, B.A. Brown and V.G. Zelevinsky, Phys. Rev. **C54** (1996) 1665.
- [34] V.V. Sokolov and V.G. Zelevinsky, Phys. Lett. **B202** (1988) 10.

- [35] S. Kamberdzhiev, J. Lisantti, P. von Neumann-Cosel, A. Richter, G. Ter-tychny and J. Wambach, Phys. Rev. **C55** (1997) 2101.
- [36] S. Ottini, N. Alamanos, A. Drouart, W. Mittig, H. Savajols, private communication.
- [37] C.Y.Wong and H.H.K. Tang, Phys. Rev. Lett. **40** (1978) 1070; Phys. Rev. **C20** (1979) 1419.
- [38] R. Balian, Y. Alhassid, and H. Reinhardt, Phys. Rep. **131** (1986) 1.
- [39] Y. Abe, S. Ayik, P. G. Reinhard and E. Suraud, Phys. Rep. **275** (1996) 49.
- [40] S. Ayik and M. Dworzecka, Phys. Rev. Lett. **54** (1985) 534; Nucl. Phys. **A440** (1985) 424.
- [41] R.H. Dicke, Phys. Rev. **93** (1954) 99
- [42] R. Broglia, T. Døssing, B. Lauritzen and B. R. Mottelson, Phys. rev. Lett. **58** (1987) 326.

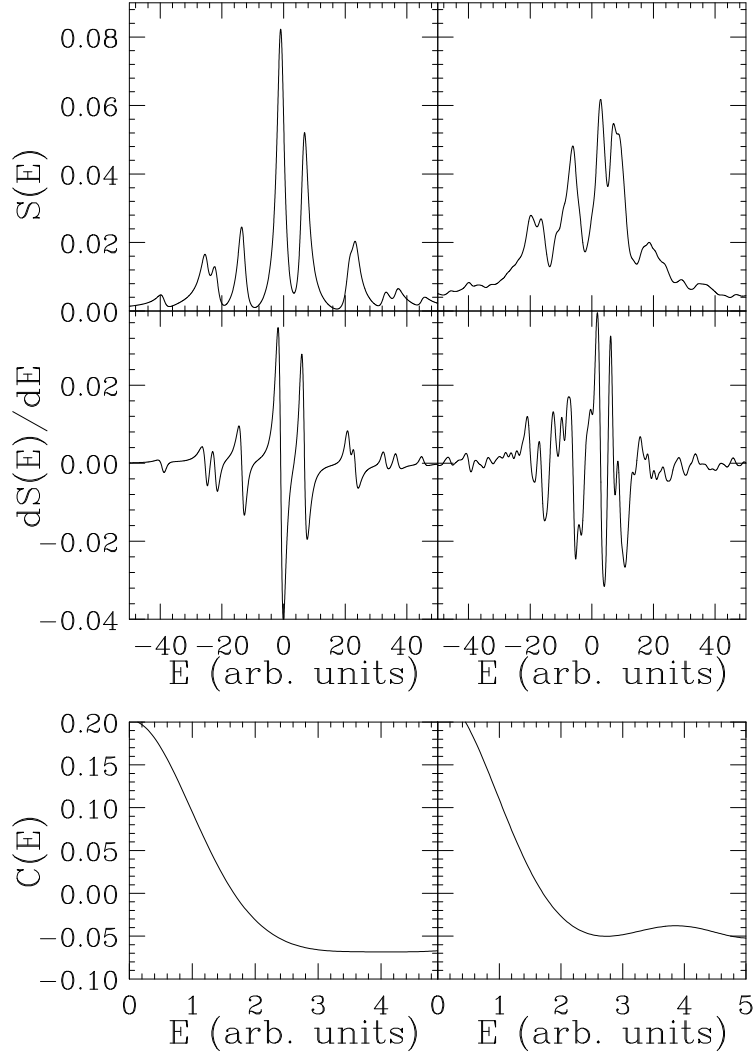


Figure 1: Top: Illustration of the strength computed for a collective excitation coupled to an ensemble of states (decay channel). In left part, few states with a width $\bar{\Gamma} = 3$ smaller than the spacing ($\Delta E \simeq 10$) is used. In right part, the opposite case of a dense spectrum of decay channels ($\Delta E \simeq 0.5$) with an average width $\bar{\Gamma} = 3$ greater than the spacing exhibit typical Ericson fluctuations. In both cases, the collective state is assumed to be spread, giving a variance around 30 in the considered interval. Middle: derivative of the strength function. Bottom : auto-correlation performed on the derivative of the strength function. In both cases, the averaged width of the decay channel $\bar{\Gamma}$ can be extracted. Note that half of the width is measured on the autocorrelation of the derivative of the strength function.

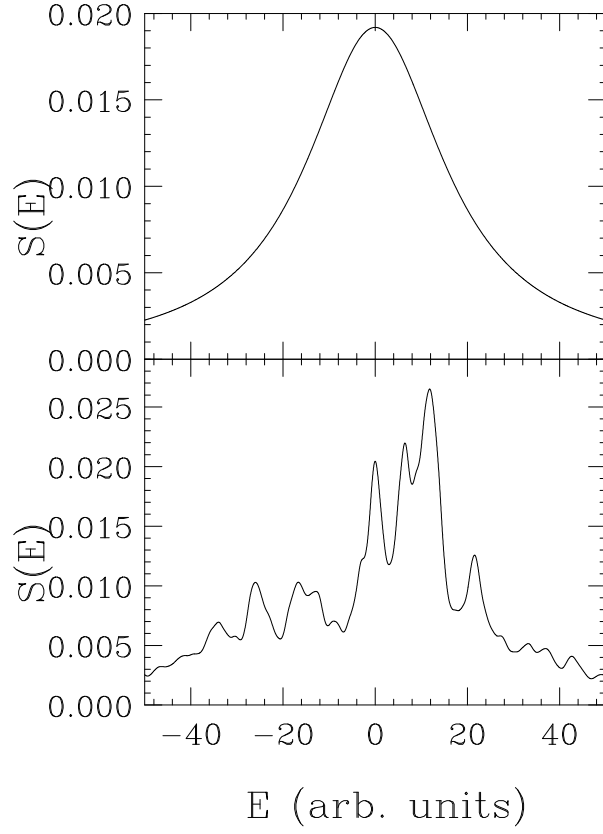


Figure 2: Illustration of the strength computed for a collective doorway state coupled to a series of decay states damped with a characteristic width larger than the spacing. Top: when the coupling matrix elements are not fluctuating. Bottom: when the residual interaction contains random terms. In both cases, the total width is roughly equal to 30 whereas the individual width and spacing are $\Gamma_{\mu_1} = 3$, $\Delta E_1 = 0.5$.

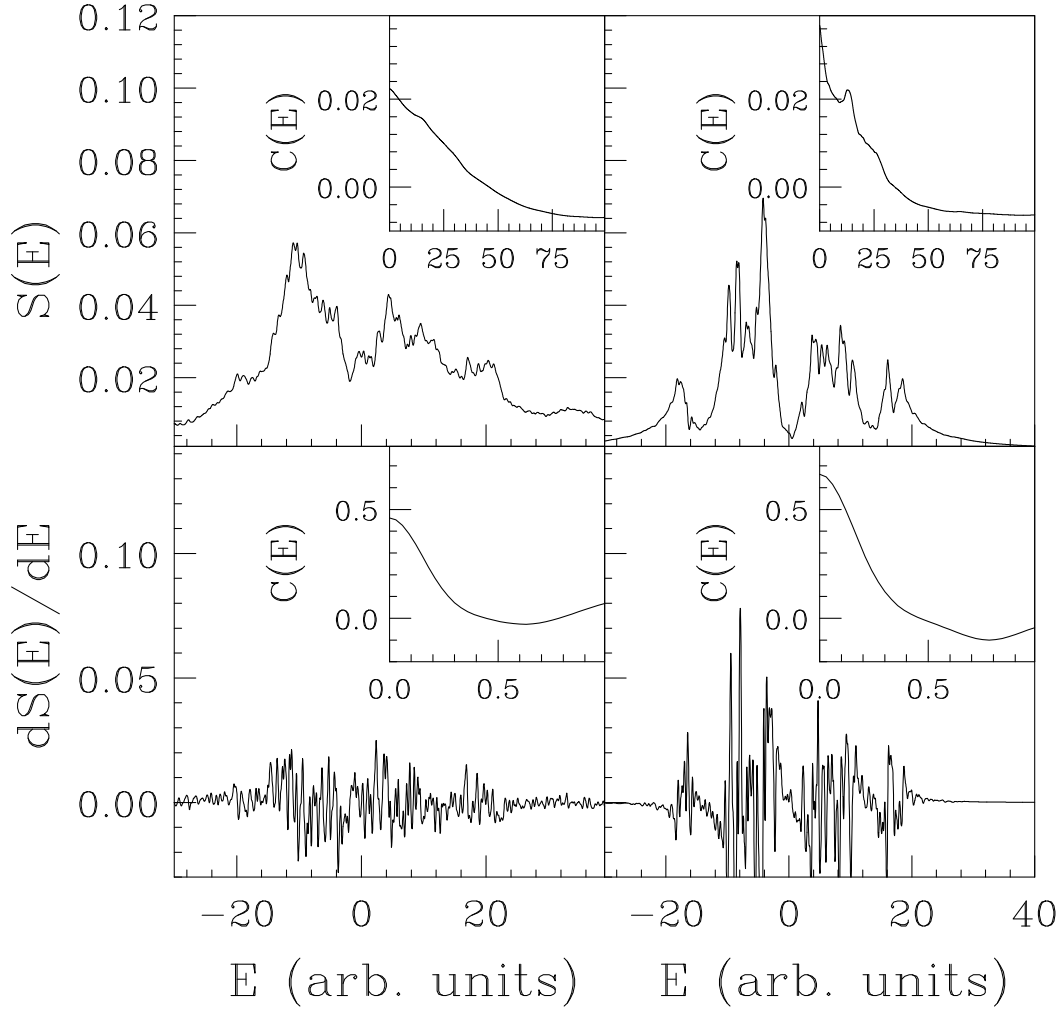


Figure 3: Top: Strength distribution presenting multiscale fluctuation. Left: Ericson fluctuations over two scales (equivalent to fig. 4). Right: Small Ericson fluctuations on top of a fragmented strength (equivalent to fig. 5). Bottom: associated derivatives of the strength distributions. In both presented cases, the total spreading width is equal to $\Gamma_{Coll} = 30$ whereas the scales of fluctuations of the two decay channels are $\Gamma_{\mu_1} = 3$ and $\Gamma_{\mu_2} = 0.5$. In each figure, the associated autocorrelation function respectively performed on the strength (top part) or on the derivative of the strength (bottom part) are presented in insert.

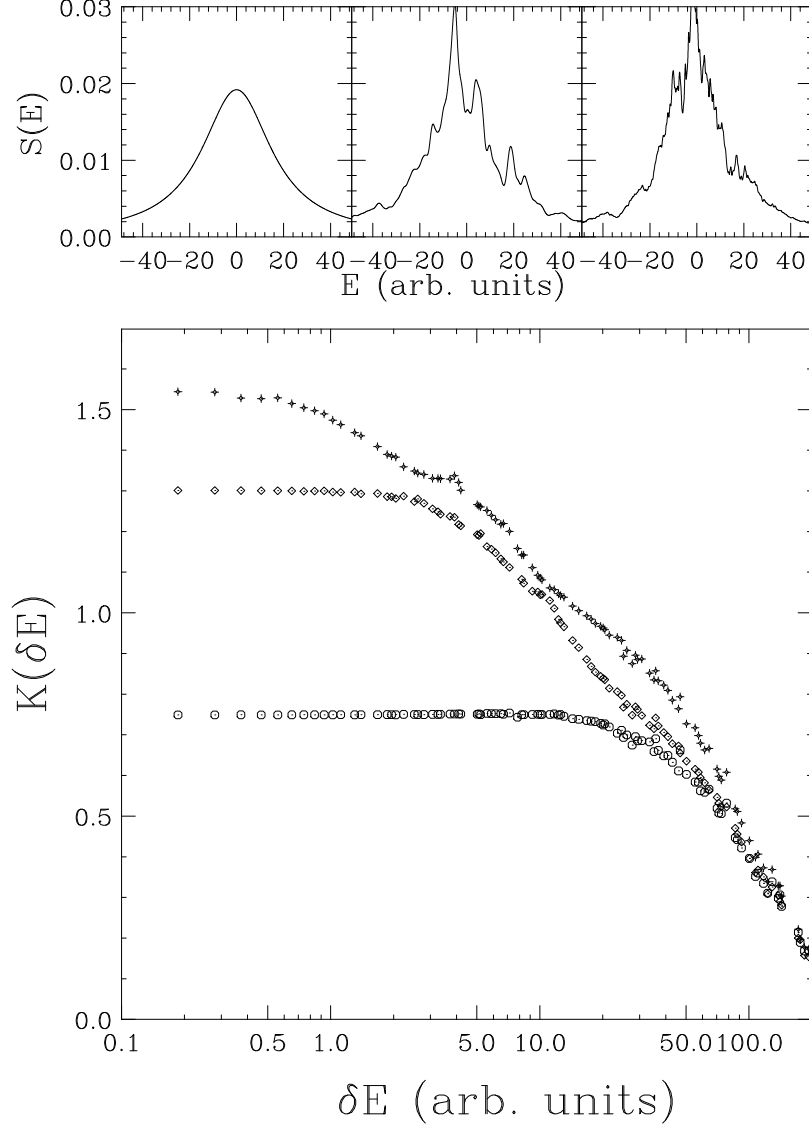


Figure 4: Top: from left to right, strength function with respectively one, two and three scales of fluctuations ($\Gamma = 30, 3$ and 0.5) are displayed. Bottom: Evolution of $K(\delta E)$ as a function of the bin size δE for the three considered strength function with one (circle), two (diamond) and three scales (cross).

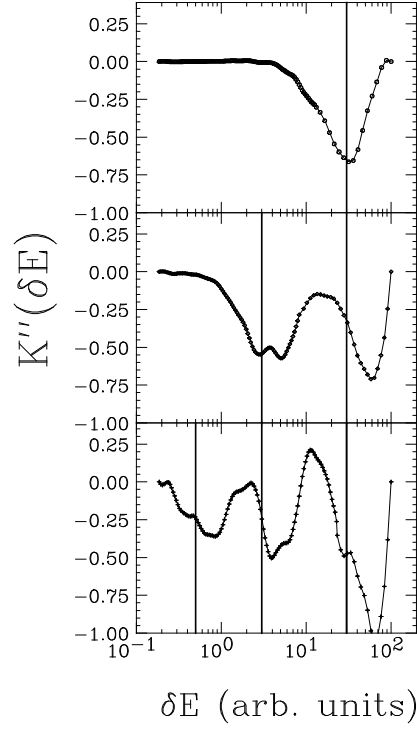


Figure 5: Numerical estimation of the second derivative (noted $K''(\delta E)$) of the entropy index as a function of the bin size δE . $K''(\delta E)$ is associated with strengths where respectively one (top), two (middle) and three (bottom) scales of fluctuations are present. In each figures, the scales of fluctuations are indicated by thick vertical lines.

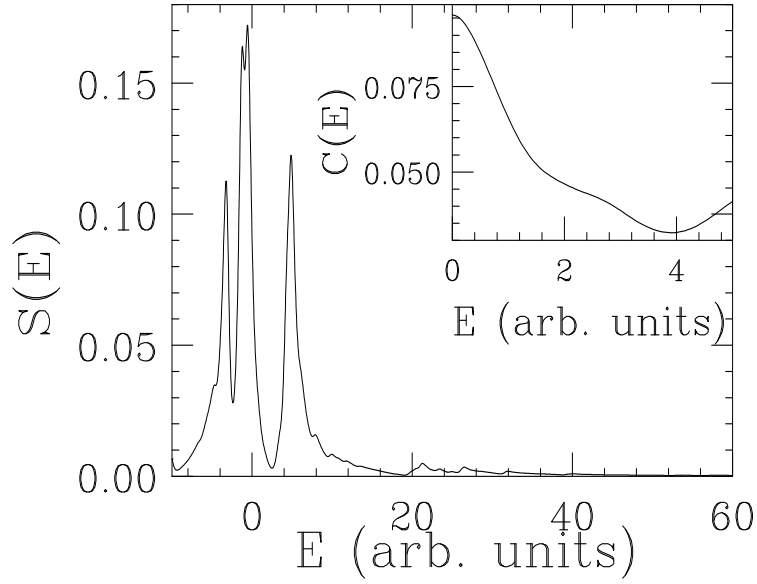


Figure 6: Illustration of the strength computed for a doorway state coupled to many decay states themselves coupled to an ensemble of states and computed introducing all types of fluctuations in the residual hamiltonian. For the calculation, we have taken $\Gamma_{Coll} = 100$, $\Delta E_1 = 3$, $\Gamma_1 = 30$ and $\Delta E_2 = 0.5$. In addition, a small width $\Gamma_{\mu_2} = 1$ has been assumed for the eigenstates of the hamiltonian in order to draw the strength function. In insert, the autocorrelation function is plotted demonstrating that the widths are proportional to ΔE_1 .